Quiz 7 Solution

February 21, 2018

1. (2 points) If $y\cos(\pi x) = x^2 + 1$, find $\frac{dy}{dx}$ at the point $(1, -2)$.

   Solution: First, take the derivative of both sides with respect to x (use implicit differentiation!):

$$\frac{d}{dx}\left[y\cos(\pi x)\right] = \frac{d}{dx}\left[x^2+1\right]$$
$$\frac{d}{dx}[y]\cdot\cos(\pi x) + y\cdot\frac{d}{dx}\left[\cos(\pi x)\right] = 2x \text{ by product rule}$$
$$\frac{dy}{dx}\cdot\cos(\pi x) + y\left(-\sin(\pi x)\cdot\frac{d}{dx}\left[\pi x\right]\right) = 2x \text{ by chain rule}$$
$$\frac{dy}{dx}\cdot\cos(\pi x) + y(-\sin(\pi x)\cdot\pi) = 2x$$

   Then, substitute $(1, -2)$ in to the equation and solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx}\cdot\cos(\pi) + (-2)(-\sin(\pi)\cdot\pi) = 2$$
$$\frac{dy}{dx}(-1) + (-2)(0) = 2$$
$$\frac{dy}{dx} = -2$$

   Answer: -2

2. (2 points) Sand is poured onto a surface at 10 ft^3/sec, forming a conical pile whose base **radius** is always equal to its height. How fast is the altitude of the pile increasing when the pile is 2 ft high? (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the base and h is the height of the cone.)

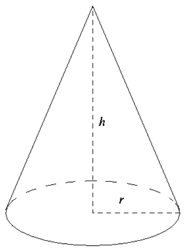
   Solution: Since $h = r$, we can rewrite V as

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (h)^2 h \\ &= \frac{1}{3}\pi h^3 \end{aligned}$$

   (This is all you needed to do for the quiz.)

Answer: $V = \frac{1}{3}\pi h^3$

Here's the full solution, in case you want to see it:

① 

② Want $\frac{dh}{dt}$ when $h = 2$ and dV$dt = 10$.

③ By the above, $V = \frac{1}{3}\pi h^3$.

④ Taking the derivative of our equation with respect to t, we get

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$$

⑤ Substituting, we get

$$10 = \pi(2^2)\frac{dh}{dt}$$

⑥ Solving for $\frac{dh}{dt}$, we get $\frac{dh}{dt} = \frac{10}{4\pi} = \frac{5}{2\pi}$.

3. (1 point) When is our exam on Thursday, March 1 at 8pm?

Answer: Thursday, March 1 at 8pm